Date - 20-07-2020

B.Sc. (Physics) Part-I, Paper-I, Group-B

Dr. S.K. Yadav ; email: sunilophy30@gmail.com

In earlier lecture notes, we have discussed the concept of coriolis force, acceleration. Here we discuss some applications of it.

$$\vec{a} = \vec{a}' + \underbrace{2\vec{\omega}\times\vec{v}'}_{\text{Coriolis acceleration}} + \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{\text{centripetal acceleration}}$$

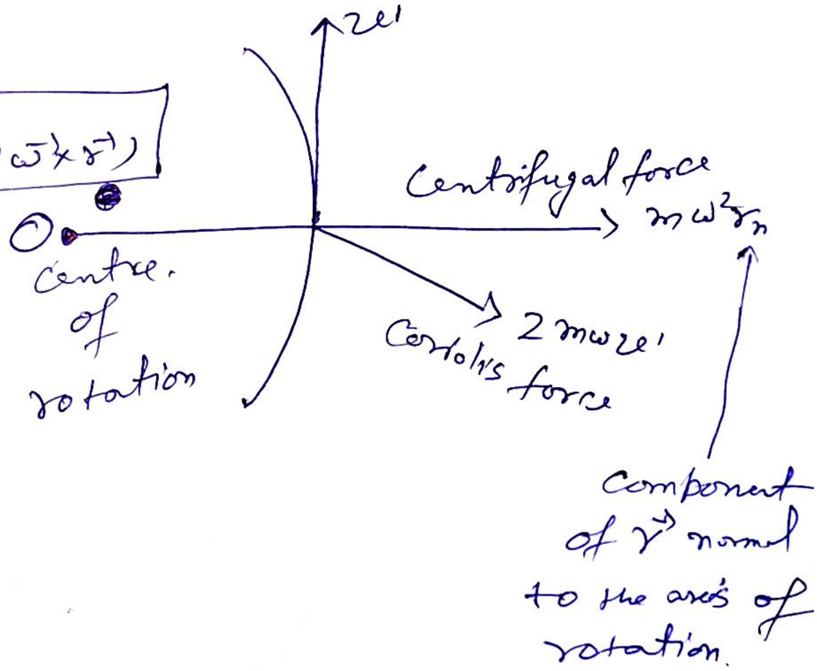
Prime on the $\vec{a}$ denotes the rotating frame.

$$\vec{F}' = \vec{F} - \underbrace{2m\,\vec{\omega}\times\vec{v}'}_{\text{Coriolis force}} - \underbrace{m\omega\times(\vec{\omega}\times\vec{r})}_{\text{centrifugal force}}$$

$$\vec{F}' = \vec{F} + \vec{F_0}$$

$$\boxed{\vec{F_0} = -2m\vec{\omega}\times\vec{v}' - m\vec{\omega}\times(\vec{\omega}\times\vec{r})}$$

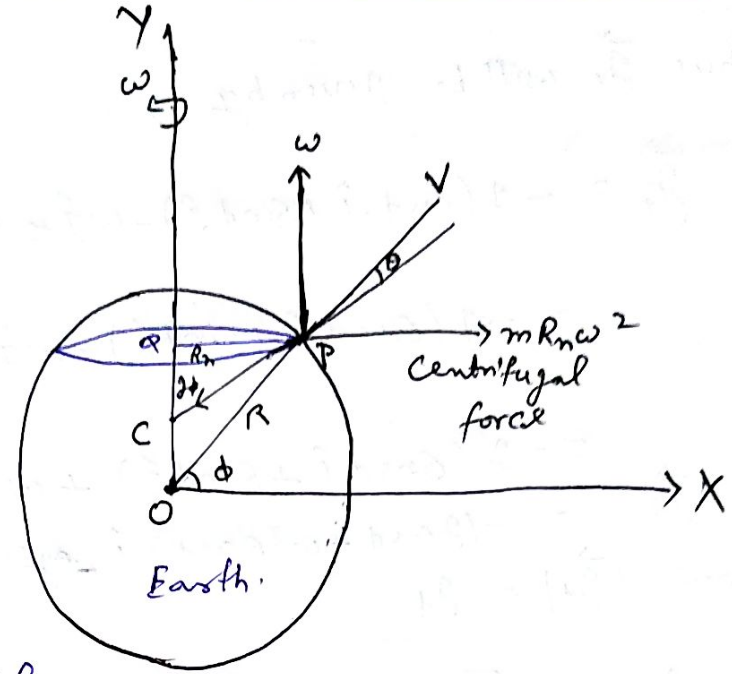
Fictitious force




Next, we will discuss some applications of it, e.g., Effect of centrifugal and coriolis force due to earth rotation.

Effect of centrifugal force :

P → s a particle at rest on the surface of the earth.




→ There will be no coriolis force acting on the particle since it is at rest in the rotating frame.

Thus the fictitious force on the particle is given ~~by~~ only by the centrifugal force which is $m\vec{\omega} \times (\vec{\omega} \times \vec{R_n})$

$\vec{R_n}$ → component of the radius R of the earth perpendicular to its axis or the distance of the particle P from the axis.

φ → latitude

g → true acceleration of the particle, $g_\phi$ → apparent acceleration

$$\vec{g_\phi} = \vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{R_n}) \quad \text{—— (1)}$$

Now if we consider that OX and OY are perpendicular and along to ω respectively. with ~~i and i~~ unit vectors $\hat{i}$ and $\hat{j}$ along OX and OY, then we can write

$$\vec{g} = -g(\cos\phi\,\hat{i} + \sin\phi\,\hat{j})$$

$$\vec{\omega} = \omega\hat{j}, \quad \text{and} \quad \vec{R_n} = R\cos\phi\,\hat{i}$$

Thus $\vec{g}_\phi$ will be given by

$$\vec{g}_\phi = -g(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) - \omega\hat{j} \times (\omega\hat{j} \times R\cos\phi\,\hat{i})$$

$$= -g(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) - \omega\hat{j} \times (-\hat{k}\,\omega R\cos\phi)$$

$$= -g(\cos\phi\,\hat{i} + \sin\phi.\hat{j}) + \omega^2 R\cos\phi\,\hat{i}$$

$$\vec{g}_\phi = -(g\cos\phi - \omega^2 R\cos\phi)\,\hat{i} - g\sin\phi\,\hat{j} \qquad (2)$$

Now $|\vec{g}_\phi| = g_\phi$

$$g_\phi = \sqrt{(g\cos\phi - \omega^2 R\cos\phi)^2 + g^2\sin^2\phi}$$

or $$g_\phi = \sqrt{g^2\cos^2\phi + \omega^4 R^2\cos^2\phi - 2\omega^2 g R\cos^2\phi + g^2\sin^2\phi}$$

$$\Rightarrow g_\phi = \sqrt{g^2 + \omega^4 R^2\cos^2\phi - 2\omega^2 g R\cos^2\phi}$$

Since $\omega$ is small $\rightarrow$ $\omega^4$ term can be neglected

$$\therefore g_\phi = \sqrt{g^2 - 2g\omega^2 R\cos^2\phi} = g\left(1 - \frac{2\omega^2 R\cos^2\phi}{g}\right)^{\frac{1}{2}}$$

or $$g_\phi = g\left[1 - \frac{1}{2}\cdot\frac{2\omega^2 R\cos^2\phi}{g} + O(\omega^4) + \dots\right.$$

taking only leading order term, i.e. upto $\omega^2$.

$$g_\phi = g\left(1 - \frac{\omega^2 R\cos^2\phi}{g}\right)$$

or $$\boxed{g_\phi = g - \omega^2 R\cos^2\phi} \qquad (3)$$

$g_\phi$ is directed towards C (See figure) instead of centre of the earth O.

Now the angle ~~of apparent~~ . $\theta$. (in the figure) which apparent direction PC makes with the true direction PO, given by.

$$\tan\theta = \frac{-(g\cos\phi - \omega^2 R\cos\phi)}{-g\sin\phi} \quad \{\text{using Eq. 2}\}$$

$$\tan\theta = \cot\phi\left(1 - \frac{\omega^2 R}{g}\right)$$

or $$\boxed{\theta = \tan^{-1}\left[\cot\phi\left(1 - \frac{\omega^2 R}{g}\right)\right]} \quad \text{— (4)}$$

$\theta$ will be maximum for $\phi = 45°$.

H.W. (1) A person weights 50 kg on the earth. Calculate the fictitious force acting on him and his effective weight in a lift when the ~~lift~~ lift is moving (i) down (ii) up, with an acceleration of 4 m/s.

(2) A reference frame a rotates with respect to another reference frame b with uniform angular velocity $\omega$. if the position, velocity and acceleration ~~that the~~ of a particle are represented by $\vec{R}$, $\vec{v}_a$, $\vec{f}_a$ respectively, Show that the acceleration of that particle in frame b is given by $\vec{f}_b = \vec{f}_a + 2\vec{\omega}\times\vec{v}_a + \vec{\omega}\times(\vec{\omega}\times\vec{R})$. Interpret this equation with reference to the motion of bodies on the earth's surface.